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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

356 **Proposed by ARTEMAS MARTIN, Ph. D., Washington, D. C.**

Solve by quadratics, if possible, the equations,

$$w(x+y+z)=a, \quad x(w+y+z)=b,$$
$$y(w+x+z)=c, \quad z(w+x+y)=d.$$

[From the *Mathematical Magazine*, Vol. II, p. 256.]

No satisfactory solution of this problem has been received.

357. **Proposed by V. M. SPUNAR, M. and E. E., Chicago, Illinois.**

Solve the system

$$\sqrt{(x^2+a^2+b^2+c^2)}=\sqrt{(y^2+b^2+c^2)}+\sqrt{(z^2+b^2+c^2)},$$
$$\sqrt{(y^2+a^2+b^2+c^2)}=\sqrt{(x^2+c^2+a^2)}+\sqrt{(z^2+c^2+a^2)},$$
$$\sqrt{(z^2+a^2+b^2+c^2)}=\sqrt{(x^2+a^2+b^2)}+\sqrt{(y^2+a^2+b^2)}.$$

Solution by ARTEMAS MARTIN, LL.D., Editor and Publisher, Mathematical Magazine, Washington, D. C.

Let $u=x^2+a^2+b^2+c^2$, $v=y^2+a^2+b^2+c^2$, $w=z^2+a^2+b^2+c^2$, then

$$x^2+c^2+a^2=u-b^2, \quad x^2+a^2+b^2=u-c^2;$$
$$y^2+b^2+c^2=v-a^2, \quad y^2+a^2+b^2=v-c^2;$$
$$z^2+b^2+c^2=w-a^2, \quad z^2+c^2+a^2=w-b^2;$$

and $x=\sqrt{(u-a^2-b^2-c^2)}$, $y=\sqrt{(v-a^2-b^2-c^2)}$, $z=\sqrt{(w-a^2-b^2-c^2)}$.

Substituting in the given equations,

$$\sqrt{u}=\sqrt{(v-a^2)}+\sqrt{(w-a^2)}\ldots(4),$$
$$\sqrt{v}=\sqrt{(u-b^2)}+\sqrt{(w-b^2)}\ldots(5),$$
$$\sqrt{w}=\sqrt{(u-c^2)}+\sqrt{(v-c^2)}\ldots(6).$$

Squaring (4), (5), and (6), transposing, etc.,

$$2\sqrt{(v-a^2)}\sqrt{(w-a^2)}=u-v-w+2a^2\ldots(7),$$
$$2\sqrt{(u-b^2)}\sqrt{(w-b^2)}=v-u-w+2b^2\ldots(8),$$
$$2\sqrt{(u-c^2)}\sqrt{(v-c^2)}=w-u-v+2c^2\ldots(9).$$

Squaring (7), (8), and (9), transposing, etc.,

$$4vw=(u-v-w)^2+4a^2u\ldots(10),$$
$$4uw=(v-u-w)^2+4b^2v\ldots(11),$$
$$4uv=(w-u-v)^2+4c^2w\ldots(12).$$

Expanding in (10), (11), and (12), transposing, etc.,

$$2uv+2uw+2vw=u^2+v^2+w^2+4a^2u \ldots (13),$$
$$2uv+2uw+2vw=u^2+v^2+w^2+4b^2v \ldots (14),$$
$$2uv+2uw+2vw=u^2+v^2+w^2+4c^2w \ldots (15).$$

From (13), (14), and (15), we have, obviously,

$$4a^2u=4b^2v=4c^2w \ldots (16),$$

which gives $v=\frac{a^2u}{b^2}$ and $w=\frac{a^2u}{c^2}$. Substituting these values of v and w in (13), we get

$$u=\frac{4a^2b^4c^4}{2a^2b^2c^2(a^2+b^2+c^2)-(a^4b^4+a^4c^4+b^4c^4)}.$$

Since $v=\frac{a^2u}{b^2}$ and $w=\frac{a^2u}{c^2}$, we have

$$v=\frac{4b^2a^4c^4}{2a^2b^2c^2(a^2+b^2+c^2)-(a^4b^4+a^4c^4+b^4c^4)},$$

$$w=\frac{4c^2a^4b^4}{2a^2b^2c^2(a^2+b^2+c^2)-(a^4b^4+a^4c^4+b^4c^4)}.$$

Substituting the above values of u, v, w in the expressions for x, y, z, we have, finally,

$$x=\sqrt{\left(\frac{4a^2b^4c^4-2a^2b^2c^2(a^2+b^2+c^2)^2+(a^2+b^2+c^2)(a^4b^4+a^4c^4+b^4c^4)}{2a^2b^2c^2(a^2+b^2+c^2)-(a^4b^4+a^4c^4+b^4c^4)}\right)},$$

$$y=\sqrt{\left(\frac{4b^2a^4c^4-2a^2b^2c^2(a^2+b^2+c^2)^2+(a^2+b^2+c^2)(a^4b^4+a^4c^4+b^4c^4)}{2a^2b^2c^2(a^2+b^2+c^2)-(a^4b^4+a^4c^4+b^4c^4)}\right)},$$

$$z=\sqrt{\left(\frac{4c^2a^4b^4-2a^2b^2c^2(a^2+b^2+c^2)^2+(a^2+b^2+c^2)(a^4b^4+a^4c^4+b^4c^4)}{2a^2b^2c^2(a^2+b^2+c^2)-(a^4b^4+a^4c^4+b^4c^4)}\right)}.$$